

4.3 Practice Questions: Brackets and Well-formed Formulas

1. "If you buy a donut at Terry's Coffee Shop, they give you a muffin and a coffee for free!"

P = "You buy a donut at Terry's Coffee Shop."

Q = "You receive a free muffin."

R = "You receive a free coffee."

$P \rightarrow (Q \ \& \ R)$

2. "You can advance the presentation by pressing spacebar or left-clicking on the mouse."

P = "You can advance the presentation."

Q = "You press the spacebar."

R = "You left-click on the mouse."

$(Q \vee R) \rightarrow P$

3. "Receiving one more piece of bad news today will be enough to make me lose my mind!"

P = "I receive one more piece of bad news today."

Q = "I lose my mind."

$P \rightarrow Q$

(Note: receiving the piece of bad news for this person is a sufficient condition for them losing their mind.)

4. "Ruthann is intelligent, funny, and very talented."

P = "Ruthann is intelligent."

Q = "Ruthann is funny."

R = "Ruthann is very talented."

$P \ \& \ (Q \ \& \ R)$

or

$(P \ \& \ Q) \ \& \ R$

5. "You can have soup, salad, or fries. You can have all three if you want."

P = "You can have soup."

Q = "You can have salad."

R = "You can have fries."

$P \vee (Q \vee R)$

or

$(P \vee Q) \vee R$

6. "Priya will go to Jerome's party as long as she finishes her logic practice and it isn't raining."

P = "Priya will go to Jerome's party."

Q = "Priya finishes her logic practice."

R = "It is raining."

$((Q \& \neg R) \rightarrow P) \& (\neg(Q \& \neg R) \rightarrow \neg P)$

or

$(Q \& \neg R) \leftrightarrow P$

7. "Applicants must possess a bachelor's degree to be qualified for the position."

P = "The applicant possesses a bachelor's degree."

Q = "The applicant is qualified for the position."

$\neg P \rightarrow \neg Q$

or

$Q \rightarrow P$

(Note: Possessing a bachelor's degree is a necessary, not a sufficient, condition, i.e. the applicant *must* have a bachelor's degree to be qualified for the position. $(\neg P \rightarrow \neg Q)$ is logically equivalent to $(Q \rightarrow P)$. Both statements tell us that whenever Q is true, P must be true as well, but they do not tell us that the truth of P is sufficient to guarantee the truth of Q.)

8. "Either it will be cloudy and raining tomorrow or the sun will shine, but not both."

P = "It will be cloudy tomorrow."

Q = "It will rain tomorrow."

R = "It will be sunny tomorrow."

$$([P \ \& \ Q] \vee R) \ \& \ \neg([P \ \& \ Q] \ \& \ R)$$

9. "No shirt, no shoes, no service."

P = "You are wearing a shirt."

Q = "You are wearing shoes."

R = "You will get service."

$$(\neg P \vee \neg Q) \rightarrow \neg R$$

or

$$R \rightarrow (P \ \& \ Q)$$

(Note: these two symbolizations are logically equivalent, i.e. they have the same truth conditions. So if one is a correct symbolization, the other is also. The first formulation mimics the form of the original statement more closely, so it's nicer; but our rule is to try to make sentences letters positive version when the English sentences are negative.)

10. "As long as Juan has free time on Saturday, hasn't twisted his ankle, and feels like doing it, he'll go for a walk in the coulees."

P = "Juan has free time on Saturday."

Q = "Juan has twisted his ankle."

R = "Juan feels like going for a walk."

S = "Juan will go for a walk in the coulees."

$$(((P \ \& \ \neg Q) \ \& \ R) \rightarrow S) \ \& \ \neg((P \ \& \ \neg Q) \ \& \ R) \rightarrow \neg S$$

or

$$((P \ \& \ (\neg Q \ \& \ R) \rightarrow S) \ \& \ \neg(P \ \& \ (\neg Q \ \& \ R)) \rightarrow \neg S)$$

or

$$((P \ \& \ \neg Q) \ \& \ R) \leftrightarrow S$$

or

$$((P \ \& \ (\neg Q \ \& \ R)) \leftrightarrow S$$

(Note: P, \neg Q, and R constitute both necessary and (jointly) sufficient conditions for Juan's going for a walk in the coulees. Necessary and sufficient conditions are also equivalent to a biconditional statement. Also note that conjunctions can only combine two conjuncts, but multiple conjunctions may be strung together with brackets to note lengthy conjunction relationships.)