## 6.4 Practice Questions: Short Truth Table Method

Test the following arguments for validity using the short truth table method:

1. 1. (P → Q)
   2. –Q
   3. ∴ –P

|  |  | ***Conclusion*** | ***Premise 2*** | ***Premise 1*** |
|---|---|---|---|---|
| **P** | **Q** | **–P** | **–Q** | **P → Q** |
| **T** | **T** | **F** | **F** | **T** |

This argument is valid. If the conclusion is false while Premise 1 is true, then Premise 2 must be false on pain of contradiction. Alternatively, if the conclusion is false and Premise 2 is true, then Premise 1 must be false.

2. 1. (P & Q)
   2. Q → R
   3. ∴ R

|  |  | ***Conclusion*** | ***Premise 1*** | ***Premise 2*** |
|---|---|---|---|---|
| **P** | **Q** | **R)** | **P & Q** | **Q → R** |
| **T** | **T** | **F** | **T** | **F** |

This argument is valid. First, we make the conclusion (R) false. Next, we set Premise 1 to true because it is a conjunction. Knowing that Premise 1 is true allows us to make P and Q true. Last, we try to make Premise 2 true. If Q is true and R is false, then Q → R must be false as well. We have already made Q true while R is false, so we cannot make Premise 2 true without contradicting ourselves.

3. 1. (P v Q)
   2. P → R
   3. ∴ Q

| P | Q | R | P v Q | P → R |
|---|---|---|---|---|
| | *Conclusion* | | *Premise 1* | *Premise 2* |
| T | F | T | T | T |

This argument is invalid. If Premise 1 is true, and Q is false, then P must be true. We may set Premise 2 to true, which results in R being true, and we have no contradiction with Q being false.

4. 1. –(P v Q)
   2. –Q v R
   3. ∴ –R

| P | Q | R | P v Q | –(P v Q) | –Q | –R | –Q v R |
|---|---|---|---|---|---|---|---|
| | | | | *Premise 1* | | *Conclusion* | *Premise 2* |
| F | F | T | F | T | T | F | T |

This argument is invalid. First we make the conclusion –R false, which makes R true. Next, we make Premise 1 true because it is a negated disjunction. When an inclusive disjunction is negated, both of its conjuncts are false. Because Q is false, –Q is true. Finally, the second premise (–Q v R) can be true: we already know that –Q is true, so the disjunction is true, whether or not R is true. Because both premises can be true while the conclusion is false, this argument is invalid

5. 1. (P & Q) → R
   2. –(–P v –Q)
   3. ∴ R

***Conclusion*** ***Premise 1*** ***Premise 2***

| **P** | **Q** | **R** | **–P** | **–Q** | **(P & Q)** | **(P & Q) → R** | **(–P v –Q)** | **–(–P v –Q)** |
|---|---|---|---|---|---|---|---|---|
| T | T | F | F | F | T | F | F | T |

This argument is valid. First, we set the conclusion to false. Next, we set Premise 2 to true; because it is a negated inclusive disjunction, we know that both disjuncts must be alse, which results in P and Q both being true. Because both P and Q are true, (P & Q) is also true. Because (P & Q) is true and R is false, Premise 1 must be set to false.

6. 1. (P v Q) & (R v S)
   2. P → –S
   3. –Q
   4. ∴ R

***Conclusion*** ***Premise 3*** ***Premise 1*** ***Premise 2***

| **P** | **Q** | **R** | **S** | **–Q** | **–S** | **P v Q** | **R v S** | **(P v Q) & (R v S)** | **P → –S)** |
|---|---|---|---|---|---|---|---|---|---|
| T | F | F | T | T | F | T | T | T | F |

This argument is valid. First we make the conclusion false. Next we make Premise 3 true, which forces Q to be false. Next we make Premise 1 true, which means both (P v Q) and (R v S) must be true. Because R is false, S must be true for (R v S) to be true. Because S must be true, –S must be false. Next, because Q is false, P must be true for (P v Q) to be true. Finally, we try to make Premise 2 true. However, we are forced to set Premise 2 to false: we already determined that P has to be true and –S has to be false when setting Premise 1 to true. Because the antecedent is true while the consequent is false, Premise 2 must be set to false. As a result, because we cannot make all of the premises true while the conclusion is false, this argument is valid.

7. 1. (P v Q) & R
   2. R → –Q
   3. ∴ P

| ***Conclusion*** | | | | | ***Premise 1*** | ***Premise 2*** |
|---|---|---|---|---|---|---|
| **P** | **Q** | **R** | **–Q** | **(P v Q)** | **(P v Q) & R** | **R → –Q** |
| F | F | T | T | F | F | T |

This argument is valid. First, we set the conclusion P to false. Next we try to make both premises true. If we make Premise 1 true, then R must be true. If R is true, and Premise 2 is true, then Q must be false. If P and Q are both false, then (P v Q) cannot be true. If (P v Q) cannot be true, then we cannot make Premise 1 true. So, at least one premise must be false when the conclusion is false.

8. 1. (P v Q)
   2. Q → R
   3. R → S
   4. –P
   5. ∴(S v T)

| | | | | | ***Premise 1*** | ***Premise 2*** | ***Premise 3*** | ***Premise 4*** | ***Conclusion*** |
|---|---|---|---|---|---|---|---|---|---|
| **P** | **Q** | **R** | **S** | **T** | **P v Q** | **Q → R** | **R → S** | **–P** | **(S v T)** |
| F | T | T | F | F | T | T | F | T | F |

This argument is valid. First, we, set the conclusion (S v T) to false. For (S v T) to be false, both S and T must be false. Next, we try to make each premise true. For –P to be true, P must be false. For (P v Q) to be true when P is false, Q must be true. Because Q is true, and Premise 2 is true, R must be true. However, if R is true and S is false, Premise 3 must be false.

9. 1. (P & Q) → R
   2. (P v S)
   3. –S
   4. (Q & T)
   5. ∴ R

| P | Q | R *Conclusion* | S | T | (P & Q) | (P & Q) → R *Premise 1* | (P v S) *Premise 2* | –S *Premise 3* | (Q & T) *Premise 4* |
|---|---|---|---|---|---|---|---|---|---|
| T | T | F | F | T | T | F | T | T | T |

This argument is valid. First, we set the conclusion R to false. Next, we try to make all the premises true. We begin with the premises that will force us to assign values to simple statements. For Premise 3 to be true, S must be false. For Premise 4 to be true, we must set Q and T to true. For Premise 2 to be true while S is false, P must be true. From what we have filled in, we can set (P & Q) to true because both P and Q are true. Finally, Premise 1 must be false: R is false while (P & Q) is true. Because at least one premise must be false when the conclusion is false, this argument is valid.

10. 1. –P
    2. Q → R
    3. (P v Q)
    4. ∴ Q & R

| P | Q | R | –P *Premise 1* | Q → R *Premise 2* | P v Q *Premise 3* | Q & R *Conclusion* |
|---|---|---|---|---|---|---|
| F | T | F | T | F | T | F |
| F | F | F | T | T | F | F |
| F | F | T | T | T | F | F |

This argument is valid. We constructed three rows to explore the three possible ways (Q & R) can be false. In each instance, at least one premise must be false when (Q & R) is false, so this argument is valid.